L'Hospital's rule

L'Hospital's Rule is used to get you out of sticky situations with indeterminate limit forms, such as $\pm\infty/\pm\infty$ or $0/0$ or $0 \cdot \pm\infty$. If you plug in the number you're approaching to the function for which you're trying to find the limit and your result is one of the indeterminate forms above, you should try applying L'Hospital's Rule.

To use it, take the derivatives of the numerator and denominator and replace the original numerator and denominator with their derivatives. Then plug in the number you're approaching. If you *still* get an indeterminate form, continue using L'Hospital's Rule until you can use substitution to get a prettier answer.

Example

Evaluate the limit.

$$\lim_{x \to 0} \frac{e^x - 1}{\sin(2x)}$$

If we try plugging in $x = 0$, we get the indeterminate form $0/0$, so we know that this is a good candidate for L'Hospital's Rule.

The derivative of our numerator is e^x. The derivative of our denominator is $2\cos(2x)$. To use L'Hospital's Rule, we take those derivatives and plug them in for the original numerator and denominator.

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$$\lim_{x \to 0} \frac{e^x}{2\cos(2x)}$$

If we now try plugging in the number we're approaching, we get a clear answer.

$$\lim_{x \to 0} \frac{e^0}{2\cos(2 \cdot 0)}$$

$$\lim_{x \to 0} \frac{e^x - 1}{\sin(2x)} = \frac{1}{2}$$

$1/2$ is our final answer. However, if plugging in 0 had resulted in another indeterminate form, we could have applied another round of L'Hospital's Rule, and another and another, until we were able to plug in the number we're approaching to get an answer that was not indeterminate.

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